## Activity: Bunko

I use the game of Bunco as the basis of a probability activity. I have used this activity for a number of years, and it serves as a good review activity before testing students on the topics of probability, discrete random variables, rules of mean and variance, and the Law of Large Numbers.

Students roll 3 dice and win $1.00 for rolling a "1", $2.00 for rolling 2 "1's", $21.00 for rolling 3 "1's", and $5.00 for rolling three of a kind of any number 2-6. Students simulate 30 trials of the game and report their results.

After gathering class-wide results, students use the rules of probability to find the actual probability distribution for the discrete random variable "$X$ = winnings". For the simulation, students are given 3 different colored dice and they record their simulation results by color of dice – this seems to help them when it comes to calculating the probabilities for the different outcomes.

Finally, students find the mean and standard deviation of the probability distribution and answer questions that involve rules of mean and variance. A worksheet that I use in class is attached, as well as a solution key for the last page of the worksheet.

If you are interested in the actual rules of Bunco, they can be found at:
http://www.worldbunco.com/rules

**AP Statistics** **Name: _____**

Activity: Bunko

**THE SET-UP**: Bunko is a game of chance played with three dice. The rules of the game are as follows:

- If you roll the dice and end up with exactly one die showing a "1", you win $1.00.
- If exactly two dice show a "1", you win $2.00.
- If all three dice show a "1", you win $21.00.
- If all three dice show the same number (any number from 2 to 6) you win $5.00.
- Any other outcome results in $0.00.

If you played the game many times, how often would you expect to win money? _____
How much would you expect to win each time you played? _____

**REQUIREMENTS:** Three dice (1 white, 1 red, and 1 green) for each pair of students.

**PROCEDURE PART I: Simulation**

1. With your partner, play a total of 30 games of Bunko. Record your results in the table below. For each game, record the results by noting the outcome of each die in order by color.

| Game # | Outcome W, R, G | $ | Game # | Outcome W, R, G | $ | Game # | Outcome W, R, G | $ |
|---|---|---|---|---|---|---|---|---|
| 1 | | | 11 | | | 21 | | |
| 2 | | | 12 | | | 22 | | |
| 3 | | | 13 | | | 23 | | |
| 4 | | | 14 | | | 24 | | |
| 5 | | | 15 | | | 25 | | |
| 6 | | | 16 | | | 26 | | |
| 7 | | | 17 | | | 27 | | |
| 8 | | | 18 | | | 28 | | |
| 9 | | | 19 | | | 29 | | |
| 10 | | | 20 | | | 30 | | |

2. Tally the number of times each dollar amount was won.

| X = $ Amount won | $0.00 | $1.00 | $2.00 | $5.00 | $21.00 |
|---|---|---|---|---|---|
| Tally | | | | | |

3. What proportion of the games did you win $1.00? _____

   What proportion of the games did you win $2.00? _____

   What proportion of the games did you win $5.00? _____

   What proportion of the games did you win $21.00? _____

   What proportion of the games did you win $0.00? _____

   On average, how much did you win each game? _____

4. Combine your results with the rest of the class and complete the table below.

| X = $ Amount won | $0.00 | $1.00 | $2.00 | $5.00 | $21.00 |
|---|---|---|---|---|---|
| Tally | | | | | |
| Relative Frequency | | | | | |

**PROCEDURE PART II: Theoretical Probability**

5. Using the rules of probability and what you have learned about discrete random variables, answer the following questions. Be sure to show work. (It may be helpful to consider the color of the dice and all possible outcomes when finding some of the probabilities.)

   (a) P(win \$21.00) =

   (b) P(win \$5.00) =

   (c) P(win \$1.00) =

   (d) P(win \$2.00) =

   (e) Use your results from above to complete the following probability distribution table for the random variable *X*.

| *X* = Bunko Winnings | \$0.00 | \$1.00 | \$2.00 | \$5.00 | \$21.00 |
|---|---|---|---|---|---|
| P(*X*) | | | | | |

   (f) How much would you expect to win each time you played the game of Bunko? How does this compare with your guess from the beginning of this activity?

   (g) What is the standard deviation for the winnings in Bunko?

   (h) Suppose the amount you win in Bunko is tripled. What are the new mean and standard deviation?

   (i) Suppose you play the game twice. What are the mean and standard deviation of your total winnings?

   (j) Suppose you play Bunko with a friend. What are the mean and standard deviation of the difference in your winnings?

   (k) How do the relative frequencies in part 4 compare to the theoretical probabilities above? Explain.

## Solution Key:

5. Using the rules of probability and what you have learned about discrete random variables, answer the following questions. Be sure to show work. (It may be helpful to consider the color of the dice and all possible outcomes when finding some of the probabilities.)

(a) P(win \$21.00) $= \frac{1}{6}\cdot\frac{1}{6}\cdot\frac{1}{6} = \frac{1}{216} = \mathbf{0.004629}$

(b) P(win \$5.00) $= \frac{5}{6}\cdot\frac{1}{6}\cdot\frac{1}{6} = \frac{5}{216} = \mathbf{0.023148}$

(c) P(win \$1.00) $= 3\cdot\frac{1}{6}\cdot\frac{5}{6}\cdot\frac{5}{6} = \frac{75}{216} = \mathbf{0.347222}$

(d) P(win \$2.00) $= 3\cdot\frac{1}{6}\cdot\frac{1}{6}\cdot\frac{5}{6} = \frac{15}{216} = \mathbf{0.069444}$

(e) Use your results from above to complete the following probability distribution table for the random variable *X*.

| *X* = Bunko Winnings | \$0.00 | \$1.00 | \$2.00 | \$5.00 | \$21.00 |
|---|---|---|---|---|---|
| P(*X*) | $\frac{120}{216}$ or $\frac{5}{9}$ | $\frac{75}{216}$ or $\frac{25}{72}$ | $\frac{15}{216}$ or $\frac{5}{72}$ | $\frac{5}{216}$ | $\frac{1}{216}$ |

(f) How much would you <u>expect</u> to win each time you played the game of Bunko? How does this compare with your guess from the beginning of this activity?

$$\mu_X = \mathbf{0}\cdot\frac{\mathbf{120}}{\mathbf{216}} + \mathbf{1}\cdot\frac{\mathbf{75}}{\mathbf{216}} + \mathbf{2}\cdot\frac{\mathbf{15}}{\mathbf{216}} + \mathbf{5}\cdot\frac{\mathbf{5}}{\mathbf{216}} + \mathbf{21}\cdot\frac{\mathbf{1}}{\mathbf{216}}$$

$$\mu_X \approx \mathbf{\$\ 0.6991}$$

(g) What is the standard deviation for the winnings in Bunko?

$$\sigma_X = \sqrt{\mathbf{0-.6991}^{\,2}\cdot\frac{\mathbf{120}}{\mathbf{216}} + \mathbf{1-.6991}^{\,2}\cdot\frac{\mathbf{75}}{\mathbf{216}} + \ldots + \mathbf{21-.6991}^{\,2}\cdot\frac{\mathbf{1}}{\mathbf{216}}}$$

$$\sigma_X \approx \mathbf{\$\ 1.6603}$$

(h) Suppose the amount you win in Bunko is tripled. What are the new mean and standard deviation?

$$\mu_{3X} = \mathbf{3}\cdot\mu_X = \mathbf{3}\cdot(\mathbf{0.6991}) \approx \mathbf{\$\,2.0972}$$

$$\sigma_{3X} = \sqrt{\mathbf{3}^2\cdot\sigma_X^2} = \sqrt{\mathbf{9}\cdot(\mathbf{1.6603})^2} \approx \mathbf{\$\,4.9810}$$

(i) Suppose you play the game twice. What are the mean and standard deviation of your total winnings?

$$\mu_{X+X} = \mu_X + \mu_X = \mathbf{0.6991} + \mathbf{0.6991} \approx \mathbf{\$\,1.3981}$$

$$\sigma_{X+X} = \sqrt{\sigma_X^2 + \sigma_X^2} = \sqrt{(\mathbf{1.6603})^2 + (\mathbf{1.6603})^2} \approx \mathbf{\$\,2.3480}$$

(j) Suppose you play Bunko with a friend. What are the mean and standard deviation of the difference in your winnings? **Let *Y* = Friend's Winnings**

$$\mu_{X-X} = \mu_X - \mu_Y = \mathbf{0.6991} - \mathbf{0.6991} = \mathbf{\$\,0.00}$$

$$\sigma_{X-Y} = \sqrt{\sigma_X^2 + \sigma_Y^2} = \sqrt{(\mathbf{1.6603})^2 + (\mathbf{1.6603})^2} \approx \mathbf{\$\,2.3480}$$

(k) How do the relative frequencies in part 4 compare to the theoretical probabilities above? Explain. **Answers will vary**