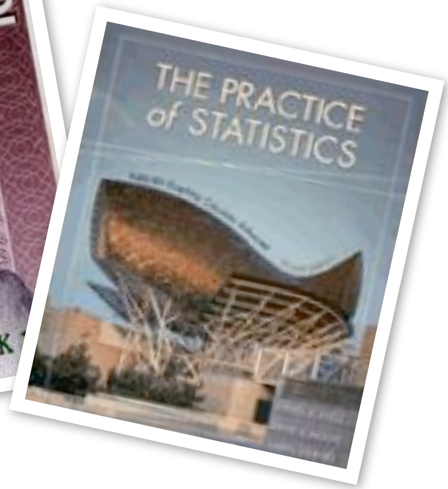
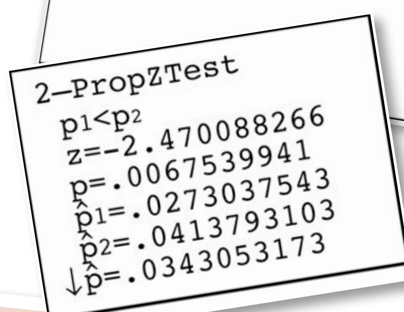


Chapter 12

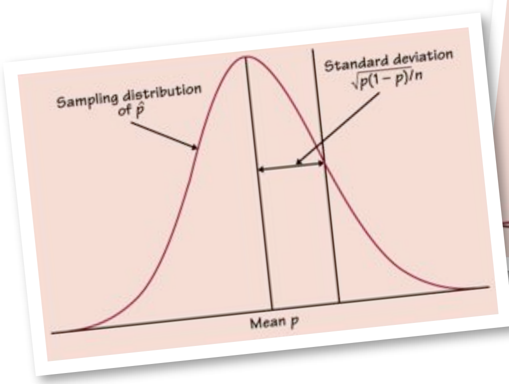
Inference for Proportions

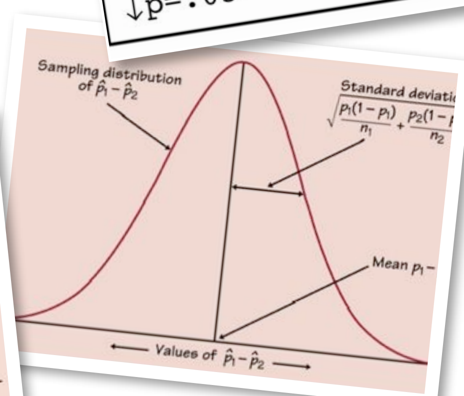
In Chapter 11, we learned inferential procedures for means. In this chapter, we will apply that logic to inference involving proportions. We will learn how to build confidence intervals and perform significance tests for one proportions as well as comparisons between two proportions.

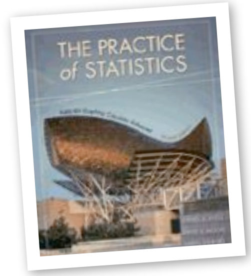


Inference for Proportions:

- 12.1 Inference for a Single Proportion
- 12.2 Comparing Proportions
- Significance Test Practice







AP Statistics Chapter 12: Inference for Proportions

"A statistical analysis, properly conducted, is a delicate dissection of uncertainties, a surgery of suppositions."
~M.J. Moroney

Tentative Lesson Guide					
Date		**Stats**	**Lesson**	**Assignment**	**Done**
Wed	2/28	12.1	Inference for Proportions	Rd 682-688, Do 1-5	
Thu	3/1	12.1	z Intervals and Tests	Rd 689-697, Do 7-9, 11, 15	
Fri	3/2	12.1	Stats Workshop	Packet Practice Problems	
Mon	3/5	**Qz**	**Quiz 12.1**		
Tues	3/6	12.2	Comparing Proportions	Rd 700-706, Do 22-24	
Wed	3/7	12.2	Significance tests	Rd 707-717, Do 26,28,30,31	
Thu	3/8	**Rev**	**Review Workshop**	Packet Practice Problems	
Fri	3/9	**Ex**	**Exam Chapter 12**	**Online Quiz Due**	

Note:
The purpose of this guide is to help you organize your studies for this chapter.
The schedule and assignments may change slightly.

Keep your homework organized and refer to this when you turn in your assignments at the end of the chapter.

Class Website:
Be sure to log on to the class website for notes, worksheets, links to our text companion site, etc.

http://web.mac.com/statsmonkey

Don't forget to take your online quiz!. Be sure to enter my email address correctly!
http://bcs.whfreeman.com/yates2e

My email address is:

jmmolesky@isd194.k12.mn.us

Chapter 12 Objectives and Skills:

These are the expectations for this chapter. You should be able to answer these questions and perform these tasks accurately and thoroughly. Although this is not an exhaustive review sheet, it gives a good idea of the "big picture" skills that you should have after completing this chapter. The more thoroughly and accurately you can complete these tasks, the better your preparation.

z-Distributions

- ☐ Describe the sampling distribution of p-hat for a single proportion or (p-hat1 - p-hat2) for a difference of proportions.
- ☐ Find z-statistics and p-values for sample proportions.

Inference for a Single Proportion

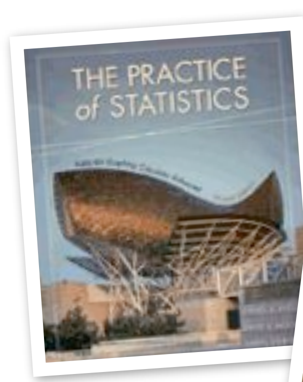
- ☐ Construct and interpret a level C confidence interval for a single proportion.
- ☐ Conduct a significance test for a claim about a single proportion.

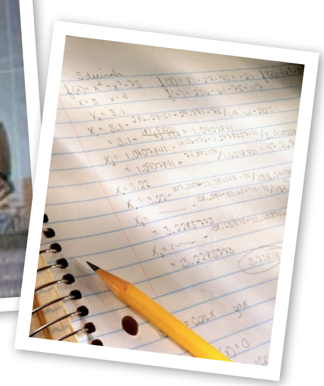
Inference for Two Means

- ☐ Describe the sampling distribution for the difference between sample proportions from two independent populations.
- ☐ Calculate and interpret a Level C confidence interval for the difference between two proportions.
- ☐ Conduct a two-sample z-test for the difference between two proportions.

Calculator Procedures

- ☐ Be able to calculate and interpret Confidence Intervals for proportions using your graphing calculator.
- ☐ Be able to perform a one- or two-sample z-test using your graphing calculator.
- ☐ Recognize that the graphing calculator is only a tool to provide you with the test statistic and p-value...it is up to YOU to interpret the results!





12.1: *Estimating a Single Proportion*

Inference Toolbox: ***Confidence Interval for a Single Proportion*** π
To Construct a Confidence Interval for a Single Proportion π**:** ☑ ☑ ☑ ☑

<u>Example</u>:

12.1: Tests of Significance for a Single Proportion

Inference Toolbox: ***Test of Significance for a Single Proportion*** π

To Test a Claim about an unknown Population Proportion π**:**

- ☑
- ☑
- ☑
- ☑
- ☑
- ☑

Example: **Are Pennies "Fair"?**

People often flip a coin to make a "random" selection between two options. However, the "randomness" depends on the assumption that the probability of getting heads is 0.5. If a penny is fair, it should be equally heavy on both sides...then it would be reasonable to assume the long-term proportion of heads would be about 0.5. We will test that assumption by testing samples of pennies to determine if they are more likely to fall heads up or heads down.

$$H_0 : \pi = 0.5$$
$$H_a : \pi \neq 0.5$$

- Stand 10 pennies on edge on a horizontal surface. Take your time...
- Hit the surface just hard enough to make the pennies fall.
- Count the number of pennies that fall heads up.
- Repeat a total of 5 times and combine your results with the class.

Trial	# Heads	Cumulative Heads	Cumulative Pennies	$\hat{p}_{heads}$
1			10	
2			20	
3			30	
4			40	
5			50	

Class Sample Proportion: **p-hat** =_________/_________=________________________

Use this proportion to test the assumption that π=0.5.

12.1: Practice

1. In the book, Life in America's Small Cities, GS Thomas reports that in 1990, 22.1% of all 16-19 year olds in Key West, Florida were high school dropouts. In 1995, a random sample of 193 people in this Key West age group showed 32 were dropouts. Does this indicate the proportion of high school dropouts in Key West is less than 22.1% Test at 5% significance.

2. The US Department of Transportation reported that 77% of all fatally injured automobile drivers were intoxicated. A random sample of 27 records in Kit Carson County, Colorado, showed that 15 involved a drunk driver. Use a 99% confidence interval to determine whether or not there is evidence that indicates the population proportion of driver fatalities related to alcohol is different than 77%.

3. A parameter of interest to car manufacturers is customer loyalty- the proportion of customers who would buy another car by that manufacturer. Suppose Chevrolet would like to estimate the true proportion of customer loyalty to within 3.4% of π at 97% confidence. How many customers would they have to survey to guarantee that accuracy?

4. In Computer Studies of the Humanities and Verbal Behavior, D Wishart and SV Leach found that 21.4% of passages in Plato's Dialogues follow a particular syllable pattern. An owner of an antiquities store in Athens claims to have a manuscript that is a part of Dialogues. A random sample of 493 passages from the manuscript showed that 136 exhibited Plato's syllable pattern. Do the data indicate this manuscript is part of Plato's Dialogues?

5. Harper's Index reported that 80% of all supermarket prices end in the digit 9 or 5. Suppose you check a random sample of 115 items in a supermarket and find 88 ending in 9 or 5. Is this evidence that less than 80% will have a price ending in 9 or 5?

6. Diltiazem is a drug commonly prescribed for hypertension but may cause headaches as a side effect. It is hypothesized regular exercise might help reduce the headaches. A random sample of 209 patients using Diltiazem exercised regularly and 16 reported having headaches. Construct a 96% confidence interval for the proportion of patients who would have headaches if they exercise.

7. Of the 38 numbers on a roulette wheel, 18 are red, 18 are black, and 2 are green. If the wheel is balanced, the probability of the ball landing on red is 18/38=.474. A gambler observes 200 spins of the wheel and finds that the ball lands on red 93 times. Do the data provide evidence at the 10% level that the ball is not landing on red the correct percentage of the time for a balanced wheel?

12.2: Inference for Two Proportions

Notation:

	Parameters		Statistics
Population	**Proportion**	**Sample Size**	**Proportion**
1			
2			

Like the one-sample procedures, two-sample procedures are based on the **Sampling Distribution** of the parameter we are testing. When comparing two proportions, it is easiest to consider the difference between them and describe the sampling distribution of the difference between the sample proportions.

"Pooled Proportion"

When comparing two proportions, we often test to determine whether or not the two population proportions are equal. Since we generally observe a difference in our p-hats, we must make an estimate as to what the two proportions may really be equal to. We call this estimate a 'pooled' proportion.

$$pooled\ \hat{p} = \frac{total\ \#\ successes}{n_1 + n_2}$$

Sampling Distribution of $(\hat{p}_1 - \hat{p}_2)$

Test Statistic for the Difference between Two Proportions

12.2: Practice

1. Many investigators have studied the effect of the wording of questions on survey responses. Consider the following two versions of a question:
   1) Would you favor or oppose a law that would require a person to obtain a police permit before purchasing a gun?
   2) Would you favor or oppose a law that would require a person to obtain a police permit before purchasing a gun, or do you think that such a law would interfere too much with the right of citizens to own guns?

463 of 615 people who were asked question 1 favored the law. 403 of 585 people who were asked question 2 favored the law. Is there evidence that the phrasing may have influenced responses?

2. Do teachers find their work rewarding? Of 395 elementary teachers surveyed, 224 indicated they were satisfied with their jobs. 126 of 266 high school teachers indicated satisfaction. Based on this data, is it reasonable to assume elementary teachers tend to be more satisfied with respect to their jobs?

3. The article, "Regrets on Early Sex" appeared in the Australian newspaper The Herald (Jan. 2, 1998). After surveying 50 males and 50 females, the authored wrote, "While 54% of women thought that they should have waited longer before having sex, only 16% of the men felt that way." Construct and interpret a 95% confidence interval for the difference in the true proportions of men and women who think they should have waited longer.

4. The article "Truth and DARE: Tracking Drug Education to Graduation" compared the drug use of 238 randomly selected high school seniors exposed to a drug education program and 335 seniors who were not exposed to such a program. Data for marijuana use follows:

	n	# Who Used Marijuana
Exposed to DARE	288	141
Not Exposed to DARE	335	181

Is there evidence at the 5% level to suggest the proportion of those who used marijuana is lower for students who were in the DARE program?

5. A person released from prison before completing the original sentence is placed under the supervision of a parole board. If that person violates specified conditions of good behavior, the board can order a return to prison. A random sample of individuals who served time in prison for either impulsive or premeditated murder produced the following data:

	Impulsive	Premeditated
n	42	40
Number with No Violation	13	22

Construct and interpret a 98% confidence interval for the difference in the proportion who successfully completed parole for impulsive and premeditated murders.

6. The positive effect of water fluoridation on dental health is well documented. Of 143 randomly selected children in a town without fluoridated water, 106 had decayed teeth. 67 of 119 children from a town with fluoridated water had decayed teeth. Is this evidence to support the claim that fluoridated water may result in fewer decayed teeth?

Hypothesis Testing in a Nutshell

A hypothesis test is used to determine whether or not a claim is true. That claim could be about a single mean value, whether or not one proportion is bigger than another, whether or not the difference between two means is zero, etc. The claim is tested by comparing a sample statistic to a hypothesized value. We call the claim we are testing the 'null hypothesis' and set up an 'alternative' based on what we are trying to prove in the problem. Our calculations are based on figuring out the probability of observing our sample value under the assumption that the null hypothesis is correct. That is, we ask ourselves, "Self, if I took many samples from a population where the null hypothesis was true, what distribution of sample values would I expect to see?" The hypothesis test places the observed value on that distribution and calculates the probability of observing it assuming the null really was true...we call that probability the P-value. If the probability is small, then we are saying "It's unlikely that I'd observe this value if the null was true...therefore, I have evidence that the null could be false." It's as simple as that.

A Hypothesis Test should consist of 4 main parts:

- ***Introduction*: Hypotheses – Null and Alternative**
- ***Conditions* for Test**
- ***Calculations* – Sketch – Test Statistic - P-value**
- ***Conclusion* - Statistical and Practical**

The test statistic formulas used to calculate the P-value all follow the same general format:

$$\text{test statistic} = \frac{\text{observed} - \text{null parameter}}{\text{standard deviation of sampling dist}}$$

One Mean	**Two Means**		**One Proportion**	**Two Proportions**
$t = \dfrac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$	$t = \dfrac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$		$z = \dfrac{\hat{p} - \pi}{\sqrt{\frac{\pi(1-\pi)}{n}}}$	$z = \dfrac{(\hat{p}_1 - \hat{p}_2)}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$
Note: If you know σ, use it in place of s and do a z-test. If you have a 'matched pairs' situation, calculate the differences and do a one sample test on those differences.			Note: $\hat{p}$ is the 'pooled' proportion.	

☑ Introduction and Hypotheses:

When you first read a problem, determine what you will be testing (means? proportions? one sample? two samples? matched pairs?). Once this is determined, set up a pair of hypotheses. The null is the claim in question and the alternative is what you think your sample is proving. Write these out in words and symbols.

☑ Conditions for the Sampling Distribution:

What you are testing determines which test statistic you will use (z-test, t-test, etc.). This test statistic will then be used to calculate the P-value...but that comes later. Choose the appropriate test based on your hypotheses and (this is important) check the conditions for that test. Remember, all of your calculations are based on a sampling distribution, so we need to make sure we satisfy the requirements for constructing that distribution!

Means	**Proportions**
• SRS • $n > 30$ or population is approx. normal	• SRS • pop > $10n$ • $n\pi > 10$, $n(1-\pi) > 10$
<u>Note</u>: To check normality, look at the sample data. Outliers and strong skewness are bad. If using matched pairs, you must indicate why.	Two sample problems require you check the assumptions for *both* samples.

☑ Calculations:

Your calculator will do most of the dirty work...your job is to interpret it. However, just because the meat-grinder does the test for you does not mean you should just copy the results from the screen to your paper. Take the time to sketch the sampling distribution (draw a normal or t distribution centered at the null value and shade in your sample observation), write the test statistic formula with the appropriate information inserted, give the z or t test value, the df (if necessary), and, most importantly, the P-value.

☑ Conclusion:

Did your test produce a small P-value (less than .01 or .05)? If so, then you have shown that it is unlikely you would observe the sample information if the null hypothesis were true...therefore, you have evidence that the null may be false! Reject or fail-to-reject the null based on your P-value and then conclude by interpreting your test in terms of the original problem. It's as simple as that!

Significance Test Practice

1. The article "Credit Cards and College Students: Who Pays, Who Benefits?" (J. College Student Development (1998): 50-56) described a study of credit card payment practices of college students. According to the authors, the credit card industry asserts at most 50% of college students carry a credit card balance month to month. However, the authors of the article report a random sample of 310 students showed 217 carried a balance month to month. Does this sample provide evidence to reject the industry claim?

2. Snake experts believe venomous snakes inject different amounts of venom when killing their prey. Researchers at the University of Wyoming studied this and reported their results in "Venom Metering by Juvenile Prairie Rattlesnakes, Crotauls v. Viridis: Effects of Prey Size and Experience," Animal Behavior (1995): 33-40. The researchers measured the average amount of venom used to kill a small mouse for 21 randomly selected inexperienced hunting snakes. Then, they measured the average amount of venom used to kill small mice for 21 experienced hunting snakes. The data follow:

	Small Mouse	
	$\bar{x}$	s
Inexperienced	3.1	1.0
Experienced	2.6	0.3

Is there a significant difference between the amount of venom used by inexperienced versus experienced snakes?

3. The article "Foraging Behavior of the Indian False Vampire Bat" (Biotropica (1991): 63-67) reported that 36 of 193 female bats in flight spent more than 5 min in the air before locating food. 64 of 168 male bats spent more than 5 min in the air. Is there sufficient evidence to suggest that the proportion of flights longer than 5 min differs for males and females?

4. An automobile manufacturer advertises that one of its models achieves 30 miles per gallon. 6 non-professional drivers are chosen at random and each drives one of the cars from Phoenix to LA. The resulting gas mileages are:

27.2 29.3 31.2 28.4 30.3 29.6

Do these data contradict manufacturer's advertisement?

5. In a study of memory recall, 8 students from a large psychology class were selected at random and given 10 min to memorize 20 nonsense words. Each was asked to list as many of the words as they could remember both 1 hour and 24 hours later. Use the following data to determine whether or not the mean number of words recalled 1 hour after memorization exceeds the mean number recalled 24 hours later by more than 3.

Subject	1	2	3	4	5	6	7	8
1 hour	14	12	18	7	11	9	16	15
24 hours	10	4	14	6	9	6	12	12