



Infinite Monkey Theorem

A monkey hitting keys at random on a typewriter keyboard for an infinite amount of time will almost surely type or create a particular chosen text, such as the complete works of William Shakespeare.

Intuitive Proof of the Theorem

The infinite monkey theorem is straightforward to prove, even without appealing to more advanced results. If two events are statistically independent, meaning neither affects the outcome of the other, then the probability of both happening equals the product of the probabilities of each one happening on its own. Suppose a typewriter has 50 keys, and the word to be typed is "banana". Typing at random, the chance that the first letter typed is b is $1/50$, as is the chance that the second letter typed is a, and so on. These events are independent, so the chance of the first six letters matching "banana" is $(1/50)^6$. For the same reason, the chance that the next 6 letters match "banana" is also $(1/50)^6$, and so on.

Now, the chance of not typing "banana" in each block of 6 letters is $1 - (1/50)^6$. Because each block is typed independently, the chance, X , of not typing "banana" in any of the first n blocks of 6 letters is $X = (1 - (1/50)^6)^n$. As n grows, X gets smaller. For an n of a million, X is 99.99%, but for an n of 10 billion X is 53% and for

an n of 100 billion it is 0.17%. As n approaches infinity, the probability X approaches zero; that is, by making n large enough, X can be made as small as one likes. If we were to count occurrences of "banana" that crossed blocks, X would approach zero even more quickly. The same argument applies if the monkey were typing any other string of characters of any length.

The same argument shows why infinitely many monkeys will (almost surely) produce a text as quickly as it would be produced by a perfectly accurate human typist copying it from the original. In this case $X = (1 - (1/50)^6)^n$ where X represents the probability that none of the first n monkeys types "banana" correctly on their first try. When we consider 100 billion monkeys, the probability falls to 0.17%, and as the number of monkeys n increases to infinity the value of X (the probability of all the monkeys failing to reproduce the given text) decreases to zero. This is equivalent to stating that the probability that one or more of an infinite number of monkeys will produce a given text on the first try is 100%, or that it is almost certain they will do so.

Probability

Ignoring punctuation, spacing, and capitalization, a monkey typing letters uniformly at random has one chance in 26 of correctly typing the first letter of Hamlet. It has one chance in 676 (26 times 26) of typing the first two letters. Because the probability shrinks exponentially, at 20 letters it already has only one chance in $26^{20} = 19,928,148,895,209,409,152,340,197,376$, roughly equivalent to the probability of buying 4 lottery tickets consecutively and winning the jackpot each time. In the case of the entire text of Hamlet, the probabilities are so vanishingly small they can barely be conceived in human terms. The text of Hamlet, even stripped of all punctuation, contains well over 130,000 letters which would lead to a probability of one in 3.4×10^{183946} . For comparison purposes, there are only about 10^{79} atoms in the observable universe.

The mere fact that there is a chance, however unlikely, is the key to the "infinite monkey theorem", because Kolmogorov's zero-one law says that such an infinite series of independent events must have a probability of zero or one. Since we have shown above that the chance is not zero, it must be one. To consider that an event this unlikely is almost guaranteed to occur given infinite time can give a sense of the vastness of infinity.

“Typing Monkeys Output Far from Great Literature”

10th May, 2003 - Jill Lawless - Associated Press

LONDON -- Give an infinite number of monkeys an infinite number of typewriters, the theory goes, and they will eventually produce the works of Shakespeare. Give six monkeys one computer for a month, and they will make a mess.

Researchers at Plymouth University in England reported this week that primates left alone with a computer attacked the machine and failed to produce a single word.

"They pressed a lot of S's," researcher Mike Phillips said yesterday. "Obviously, English isn't their first language."

In a project intended more as performance art than scientific experiment, faculty and students in the university's media program left a

computer in the monkey enclosure at Paignton Zoo in southwest England, home to six Sulawesi crested macaques.

Then, they waited.

At first, said Phillips, "The lead male got a stone and started bashing the hell out of it.

"Another thing they were interested in was in defecating and urinating all over the keyboard," added Phillips, who runs the university's Institute of Digital Arts and Technologies.

Eventually, monkeys Elmo, Gum, Heather, Holly, Mistletoe and Rowan produced five pages of text, composed primarily of the letter S. Later, the letters A, J, L and M crept in.

The notion that monkeys typing at random will eventually produce literature is often attributed to Thomas Huxley, a 19th-century scientist who supported Charles Darwin's theories of evolution. Mathematicians have also used it to illustrate concepts of chance.

The Plymouth experiment was funded by England's Arts Council and part of the Vivaria Project, which plans to install computers in zoos across Europe to study differences between animal and artificial life.

Phillips said the results showed that monkeys "are not random generators. They're more complex than that.

"They were quite interested in the screen, and they saw that when they typed a letter, something happened. There was a level of intention there."

“If the monkey could type one keystroke every nanosecond, the expected waiting time until the monkey types out Hamlet is so long that the estimated age of the universe is insignificant by comparison ... this is not a practical method for writing plays.” - Gian-Carlo Rota