APSTATS

A *Fabulous* PROBABILITY CASINO LAB

AP Statistics Casino Lab: Instructions

The purpose of this lab is to allow you to explore the rules of probability in the setting of real-life games. You will get a chance to simulate playing several casino-type and other popular games of chance. Your task is to collect information/data about each game and answer the corresponding questions.

Random variables are the raw material for statistical inference. We have been studying random variables for some time, but we made them the center of our attention in Chapter 7. While an individual outcome is unpredictable for a given random variable, probability theory tells us that there is a certain type of regularity or predictability in the results.

Casinos rely on the laws of probability and expected values of random variables to guarantee profits. While some individuals walk away quite wealthy, most leave a little (or a lot) less wealthy than when they entered.

Casino stations are set up around the room. Feel free to move from station to station throughout the hour. However, be sure to play each game at least once. You must collect data from each station in order to answer the questions.
Try your hand at the following stations:

- **Station 1: Craps**
- **Station 2: Blackjack**
- **Station 3: Roulette**
- **Station 4: Monte's Dilemma**
- **Station 5: Coins, Dice, Cards**

Station 1: The Game of Craps

Adapted from Yates, Moore, Starnes "The Practice of Statistics" Resources

1) Roll a pair of dice.
   a. If the sum is 7 or 11, tally a WIN and start a new game.
   b.If the sum is 2, 3, or 12, tally a LOSE and start a new game.
   c.If the sum is anything else, mark it down as your "Point" and roll again.
   - If you roll your "Point" value, tally a WIN and start a new game.
   - If you roll a 7, tally a LOSE and start a new game.
   - If you roll any other number, continue rolling.
2) Play 20 games total and record your win/lose data.

Game	Point	Rolls	W/L
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			

Game	Point	Rolls	W/L
11			
12			
13			
14			
15			
16			
17			
18			
19			
20			

In what proportion of games did you win on your first roll? _____________

Theoretically, what proportion of games would result in a win on the first roll?_______________

In what proportion of games did you lose on your first roll?_______________

Theoretically, what proportion of games would result in a loss on the first roll?_______________

In what proportion of games did you actually win?_______________

Craps Probability Questions:

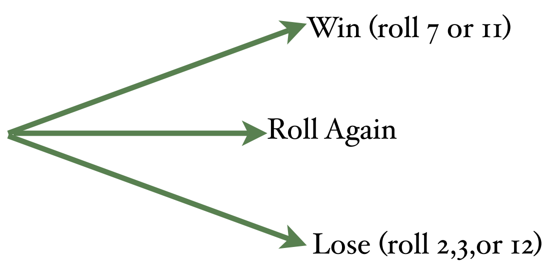
1. What is the probability that you would obtain a sum of 7 or a sum of 11 on the first roll?

2. What is the probability that you would obtain a sum of 2,3,or 12 on the first roll?

3. What is the probability that you would roll again after your first roll?

4. Suppose you roll a sum of 8 on your first roll. This establishes your point=8. Find the probability you subsequently win the game, given your point is 8. That is, What is P(win|8)?
{hint: consider the fact that the only outcomes that now matter are rolling a 7 and rolling an 8}

5. Complete the Tree Diagram below for the basic game of craps:



6. Find the P(win) in the game of craps. {hint: P(win)=P(win on first roll)+P(win on subsequent rolls)}

Station 2: Blackjack

Adapted from Yates, Moore, Starnes "The Practice of Statistics" Resources

Blackjack, also known as twenty-one, is one of the most popular casino card games in the world. Much of blackjack's popularity is due to the mix of chance with elements of skill, and the publicity that surrounds card counting (keeping track of which cards have been played since the last shuffle).

When blackjack was first introduced in the United States it was not very popular, so gambling houses tried offering various bonus payouts to get the players to the tables. One such bonus was a 10-to-1 payout if the player's hand consisted of an ace and a black Jack (either the Jack of clubs or the Jack of spades). This hand was called a "blackjack" and the name stuck to the game. As the game is currently played, a "blackjack" may not necessarily contain a jack at all. (Source: Wikipedia)

Basic Rules: To play the game, each player is dealt 2 cards face-up. The dealer also receives 2 cards, one face-down and one face-up. The goal is to have a point total as close to 21 as possible, or at least greater than the dealer's total. Players may choose to "hit" or get another card to raise their point total. However, having a total greater than 21 results in a "bust" and the player loses the hand. Cards are worth face value except face cards which are worth 10 points and the Ace, which is worth 1 or 11.

Let's consider some probabilities for the dealer's hand. Assume we are dealing without replacement from a single, well-shuffled deck.

1. Deal a hand, check the result, and record whether or not it was a true blackjack, or twenty-one. Repeat 10 times.

"Blackjacks"_____________ # "Twenty-ones"__________________

2. Suppose one card is dealt face-down and one card is face-up. Given that the face-up card is an ace...

P("Blackjack"|ace)=___________ P("Twenty-one"|ace)=_____________

3. Suppose the face-up card is a black jack (spades or clubs). Find...

P("Blackjack")|black jack)=___________ P("Twenty-one"|black jack)=____________

4. What is P("Blackjack")?

5. What is P("Twenty-one") with two-cards?

8. Are the events A=face-up card is a black jack and B=get a true "Blackjack" independent? Show work.

7. In many casinos, the dealer must hit until he or she has at least 17. If the dealer's face-up card is a 10 or Face Card, what is P(dealer points $\geq$ 17)?

Station 3: Roulette

Adapted from Yates, Moore, Starnes "The Practice of Statistics" Resources

Roulette is a French word meaning "small wheel". To play the game, a "croupier" turns a round roulette wheel which has 38 separately numbered pockets in which a ball must land. The main pockets, numbered from 1 to 36, alternate between red and black, but the pockets are not in numerical order around the wheel, and there are instances of consecutive numbers being the same color. There is also a green pocket numbered 0 and a second green pocket marked 00. If a player bets on a single number and wins, the payout is 35:1, meaning that for each dollar bet, one wins an additional $35.00.

The typical coloring scheme for the numbers in US casinos is listed below:

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**
R	**B**	**R**	**B**	**R**	**B**	**R**	**B**	**R**	**B**	**B**	**R**	**B**	**R**	**B**	**R**	**B**	**R**	**R**
20	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**	**34**	**35**	**36**	**0**	**00**
B	**R**	**B**	**R**	**B**	**R**	**B**	**R**	**B**	**B**	**R**	**B**	**R**	**B**	**R**	**B**	**R**	**G**	**G**

You can simulate a roulette wheel by selecting a random integer on your calculator. With a partner, use RandInt(1,38) to simulate the wheel {37=0 and 38=00}. Guess "red" or "black" and have your partner 'spin the wheel'. Note whether or not you win. Repeat 10 times.

Spin	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Guess										
W/L										

Suppose you bet $1 to play a game. If you guess "red" or "black", and are correct, you win $1 and get your original dollar back. If you're wrong, you lose the $1 bet. Let X = the amount gained on a single play. Complete the probability distribution for X.

X		
P(X=x)		

Find the Expected Gain per play.

Calculate the Standard Deviation of the amount gained per play.

Suppose you play two games. What are the expected gain and standard deviation of the total amount gained for the two games?

Bonus: One strategy is to always bet "red". What is P(red)? Define X=#red in 38 spins. What is the probability you'll break even by betting on red-only in 38 spins? That is, what is P(19 red in 38 spins)?

Station 4: Monte's Dilemma

Adapted from Yates, Moore, Starnes "The Practice of Statistics" Resources

This game is based on the old television show "Let's Make A Deal", hosted by Monte Hall. At the end of each show, the contestant who had won the most money was invited to choose from among three doors: Door #1, Door#2, or Door#3. Behind one of the three doors was a very nice prize. But behind the other two were rather undesirable prizes – say goats. The contestant selected a door. Then, Monte revealed what was behind one of the two doors that the contestant DIDN'T pick – goats. He then gave the contestant the option of sticking with the door she had originally selected or switching to the other, unrevealed, door.

The question is, is it better to stick with your original guess, or should you switch to the remaining door?

Simulate the game 20 times. There are 3 cards at this station, the "Ace" represents a nice prize, while the two "Jacks" represent goats. Have a partner act as the host and arrange the three cards behind the three 'doors' at the table. Pick a door. Your partner should open one of the doors you didn't pick, revealing a Jack. You must then decide to Stick or Switch. Pick a strategy and use it on 10 trials. Record your results. Then, change strategies and use it on 10 trials.

Always "Stick" Strategy		
Trial	**Door Chosen**	**Win/ Lose**
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		

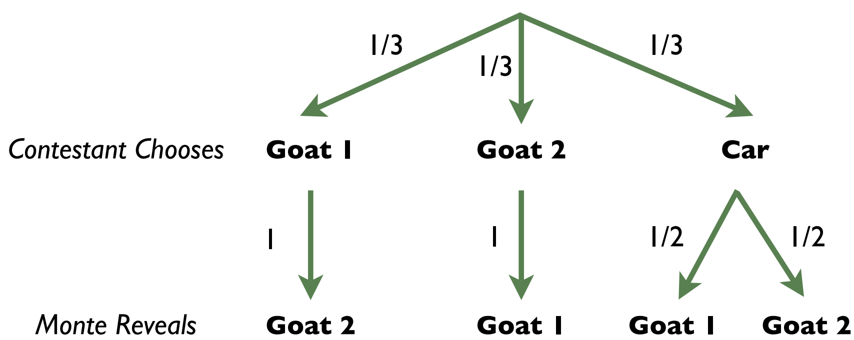
Always "Switch" Strategy		
Trial	**Door Chosen**	**Win/ Lose**
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		

1. Calculate the % of wins in each strategy. Does there appear to be a better strategy?

2. What is the probability you would have picked the nice prize on the first selection?

3. Intuition tells you it shouldn't matter whether you stick or switch...the P(nice prize) is still 1/3, right? Or is it? Do you agree or disagree? Why?

4. Consider the following tree diagram. Use it to calculate the P(each outcome):



P(outcome)				
Stay	**Goat 1**	**Goat 2**	**Car**	**Car**
Switch	**Car**	**Car**	**Goat 2**	**Goat 1**

Regardless of what you originally choose, what is P(Car) if you Stay?

Regardless of what you originally choose, what is P(Car) if you Switch?

Is it better to Switch or to Stay?

Bonus 1: A woman and a man (who are unrelated) each have two children. At least one of the woman's children is a boy, and the man's older child is a boy. Which is more likely: that the man has 2 boys or that the woman has 2 boys?

Bonus 2: There are 3 prisoners, A, B, and C. Two of them will be released and one will be executed. "A" asks the warden to tell him the name of one of the others in his cohort who will be released. As the question in not directly about "A's" fate, the warden obliges and says, truthfully, "B will be released."

"A" is offered the choice of switching fates with "C"...should he?

Station 5: Coins, Dice, Cards

Adapted from Yates, Moore, Starnes "The Practice of Statistics" Resources

In this game, begin by tossing 4 coins simultaneously. Count the number of heads. If the number is 0, you lose. If this number is *even*, then you draw a card from the deck. If the number is *odd*, then you roll a pair of dice. You win the amount shown on sum the dice or card denomination (A=1, J=11, Q=12, K=13)

Trial	# Heads	Card/ Dice Sum	Winnings
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			

1. Play this game 10 times and record your results.

2. What is P(10 on dice after tossing an odd # heads).

3. What is P(Draw 10 after tossing an even # of heads).

4. What is the expected winnings for this game? What would the operator have to charge to make it a fair game? You may wish to fill in the tables below to help you out...

# Heads	0	1	2	3	4
P(X=x)					

Sum	2	3	4	5	6	7	8	9	10	11	12
P(X=x)											

Card	1	2	3	4	5	6	7	8	9	10	11	12	13
P(X=x)													

Winnings	0	1	2	3	4	5	6	7	8	9	10	11	12	13
P(X=x)														