NonLinear Transformations

This activity can be used to introduce the concept of transforming nonlinear data to achieve linearity. The class will witness the "decay" of m&m's by shaking a container of the candies and removing those that don't display an "m". The activity continues until all m&m's have "decayed", at which point the data is plotted and discussed, leading to an introduction of transformations.

Materials Needed:

- One 1.69oz bag of m&m's - not *peanut*...the candies must be flat
- A container with a cover to shake the candies

Activity:

Discuss the decay of radioactive materials. Students will, most likely, be familiar with exponential decay and the characteristics of the graphs of exponential function. Note that we have been dealing primarily with linear growth and decay, but many situations involve nonlinear patterns. This chapter will introduce us to methods that transform nonlinear data to more linear patterns that can be modeled using the method of least squares regression learned previously.

Since we can't use *actual* radioactive isotopes, we'll simulate the decay using a package of m&m candies. When the candies are mixed up, an "m" up indicates an active candy, while an "m" down indicates a decayed one. Start by counting and placing the m&m's in a container such that all of the candies fit in one layer. Note this number as the number of active candies in round 1. Pass the container to a student and instruct them to shake it and then remove the decayed candies, reporting how many active candies remain. Continue until all candies have "decayed".

Plot the (round, active) data and note the strength, direction, and form. You should have a nonlinear decay form, so modeling with a linear function may not be the most appropriate. Walk the students through the idea of transforming data to achieve linearity. Start by calculating log(round) and log(active). Then, plot (round, log(active)) and note the strength, direction and form. Next, plot (log(round), log(active)) and again note the strength, direction, and form. Students should note that these transformations resulted in somewhat more linear scatterplots.

Choose the "most linear" of the scatterplots and find the Least Squares Regression Line. Discuss the form of this prediction model...is it in the terms of the original data? If not, can we transform the equation to a more useful model?

This activity serves as an overview of the concept of transforming data to achieve linearity. It is not meant to highlight all of the details of transformations. A detailed study of transformations and nonlinear modeling should follow during which you can discuss other transformations, how to "back transform", etc.

"Moleskium" Decay – Nonlinear Transformation Activity

The decay of radioactive isotopes typically creates a nonlinear relationship between time and amount remaining. "Moleskium" is a little-known element found in a short, highly caffeinated math teacher at LSHS. Without proper caffeination, "Moleskium" can decay quite rapidly...trust me, it's not a pretty site when that happens.

We will be simulating the decay of "Moleskium" through the use of m&m candies. As we proceed through this simulation, be sure to dispose of the "decayed" m&m's in a nearby oral cavity or waste-disposal container.

Disclaimer: DO NOT eat *actual* radioactive isotopes. These are *pretend* so it's ok.

"Moleskium" Decay Simulation:

- ☑ Start with a sizeable (n>60) quantity of m&m's in a covered container.
- ☑ Shake the container to simulate typical Mr.M. activity.
- ☑ Open the container and remove all m&m's that have "decayed" {ie, no "m" showing}
- ☑ Count the remaining m&m's and record in the table below.
- ☑ Repeat until all m&m's have "decayed".

Round Number	Isotopes Remaining
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	

☑ Enter the "Round" into L1 and "Isotopes" into L2.

☑ Display the relationship with a scatterplot and interpret in context:

☑ Use your calculator to find the LSRL for (round, isotopes):

LSRL:____

☑ Sketch the residual plot for this LSRL and use it to discuss the appropriateness of using a linear model for this relationship.

"Moleskium" Decay Activity - Continued

We can tell by the scatterplot of (round, isotopes) and by the residual plot of (round, residual) that a nonlinear model may be a more appropriate model for this situation. However, the question that remains is, "Which model is more appropriate? Exponential or Power?" Our eyes are fairly good at judging whether or not points lie on a straight line. However, they are not very good at spotting the difference between power and exponential curves. Therefore, we'll use the fact that certain transformations of power and exponential data produce linear relationships. If we can determine which transformation does a better job of "straightening out the data" then we'll have a better idea which model is best.

☑ Use your list editor to define L3 = log(round) and L4 = log(isotopes).

Remember, if (x, logy) is linear, an **exponential** model may be best. If (logx, logy) is linear, a **power** model may be best. All we have to do is plot the two and determine which is "more linear".

☑ Sketch (x, logy) and interpret	☑ Sketch (logx, logy) and interpret

☑ Find the LSRL for the transformed data that exhibits the most linear relationship. Justify "most linear" by considering the correlation and residual plots for each set of transformed data.

☑ Since the LSRL is written in terms of transformed data, transform it back into the original terms of the problem. That is, find a prediction model that will convert "round number" into "isotopes remaining".

Final Prediction Model for "Moleskium" Decay:____________________________