Goodness of Fit

{Adapted from "The Practice of Statistics, 2e by Yates, Moore, Starnes}

This activity can be used to introduce the concept of Chi-square distributions and the "Goodness of Fit" test. Using a bag of m&m's, you will compare the observed color counts to the expected color counts as noted by the m&m/Mars company.

Materials Needed:

- At least one 1.69oz bag of *milk chocolate* m&m's

Activity:

Read the statement from the m&m/Mars company Consumer Affairs Department and discuss what you would expect to see in a bag of m&m's if the statement is true. Open a bag and count the total number of candies (do not separate by color...yet). Using the total number of candies, have students calculate the expected number of each color based on the company's claim. Have them enter these numbers on their table.

Count each color and report these values to the students to enter on their tables. Discuss any differences or extreme values. Based on these counts, do we have evidence to suspect the claim may not be true?

Introduce the class to the concept of a "Goodness of Fit" test in which we can test how well an entire distribution of counts fits a hypothesized distribution. For each color, have students calculate $(Observed - Expected)^2/Expected$. Note any extreme values.

If the sample distribution does not vary much from the claimed proportions, these values should be close to zero. As the sample deviates more and more from the hypothesized distribution, these values will increase. The more extreme these values are, the less likely the sample is to be from the hypothesized distribution. Calculate the Chi-square test statistic by adding up all of the component values and determine the likelihood of observing a bag of m&m's with this color distribution assuming the company's claim is true.

m&m/Mars Company Consumer Affairs Department Announcement

"On average, the new mix of colors of m&m's plain chocolate candies will contain 30 percent browns, 20 percent yellows and reds, and 10 percent each of oranges, greens, and blues. While we mix the colors as thoroughly as possible, the above ratios may vary somewhat, especially in the smaller bags. This is because we combine the various colors in large quantities for the last production stage (printing). The bags are then filled on high-speed packaging machines by weight, not by count."

Did I Get Enough Blue m&m's?
Chi-Square Goodness of Fit Calculations

According to the m&m/Mars company, in 1995 *"...the new mix of colors of m&m's plain chocolate candies will contain 30 percent browns, 20 percent yellows and reds, and 10 percent each of oranges, greens, and blues."* However, the mix of colors has been known to change every few years. Your task today is to determine whether or not the current mix of colors matches that of 1995. We want to see if there is sufficient evidence to reject the company's 1995 claim. To do this, we'll be introduced to a new type of test -- the Chi-square Goodness of Fit Test.

- Open a bag of milk chocolate m&m's and carefully count how many of each color are in the sample. (Or, use the data from your teacher's bag) Record the observed data in the "observed" row of the table below.
- Using the statement from the m&m/Mars company, determine how many of each color you expected to see. Note, you'll have to figure this out using the total number of m&m's in your or your teacher's sample bag. Enter these counts in the "expected" row below.

	Brown	Yellow	Red	Orange	Green	Blue	Total
Observed							
Expected							
$(O-E)^2/E$							

If your bag reflects the distribution advertised in 1995, there should be little difference between the observed and expected counts. To quantify the difference, we'll calculate a total which we'll call "Chi-Square" or X^2.

- For each color, perform this calculation: (observed - expected)^2 / expected. Enter each value in the last row of the table. Add up all of these "component" values to find X^2.
- If this total value is small, we have little evidence to suggest a difference in distributions. However, the larger X^2 gets, the more evidence we have to suggest the company's claim may no longer be applicable to bags of milk chocolate m&m's.

To determine the likelihood of observing a difference between observed and expected as extreme as the one we observed, we must look up the p-value on a Chi-square table. Chi-square distributions are skewed right and specified by degrees of freedom. In a Goodness of Fit test, the degrees of freedom equal one less than the number of categories.

Find the p-value for our test by looking up X^2 for 5 degrees of freedom. Sketch the curve and observed X^2 below. Interpret the result in the context of the problem.