"FRAPPY"
{Free Response AP Problem...Yay!}

2006
Problem 6

The following problem is taken from an actual Advanced Placement Statistics Examination. Your task is to generate a complete, concise statistical response in 25 minutes. You will be graded based on the AP rubric and will earn a score of 0-4. After grading, keep this problem in your binder for your AP Exam preparation.

A manufacturer of thermostats is concerned that the readings of its thermostats have become less reliable (more variable). In the past, the variance has been 1.52 degrees Fahrenheit (F) squared. A random sample of 10 recently manufactured thermostats was selected and placed in a room that was maintained at 68 degrees F. The readings for those 10 thermostats are given in the table below.

Thermostat	1	2	3	4	5	6	7	8	9	10
Temperature (°F)	66.8	67.8	70.6	69.3	65.9	66.2	68.1	68.6	67.9	67.2

Scoring:

(a) State the null and alternative hypotheses that the manufacturer is interested in testing.

It can be shown that if the population of thermostat temperatures is normally distributed, the sampling distribution of $\dfrac{(n-1)s^2}{\sigma^2}$ follows a chi-square distribution with $n-1$ degrees of freedom.

(b) Calculate the value of $\dfrac{(n-1)s^2}{1.52}$ for these data.

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(c) Assume that the population of thermostat temperatures follows a normal distribution. Use the test statistic $\dfrac{(n-1)s^2}{1.52}$ from part (b) and the chi-square distribution to test the hypotheses in part (a).

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(d) For the test conducted in part (c), what is the smallest value of the test statistic that would have led to the rejection of the null hypothesis at the 5 percent significance level?

Mark this value of the test statistic on the graph of the chi-square distribution below. Indicate the region that contains all of the values that would have led to the rejection of the null hypothesis.

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Chi-square values
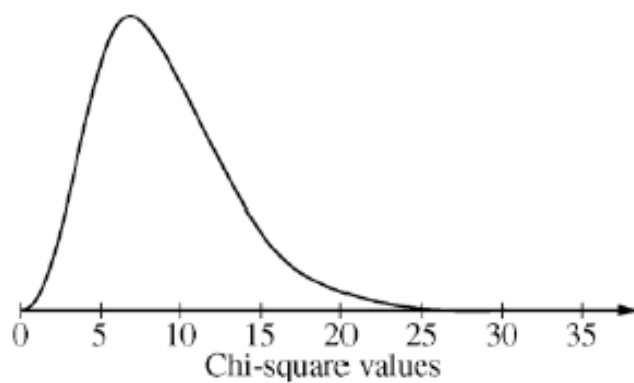
(e) Using simulation, 1,000 samples, each of size 10, were randomly generated from 3 populations with different variances. Each population was normally distributed with mean 68 and variance greater than 1.52.

The histograms below show the simulated sampling distribution of $\dfrac{(n-1)s^2}{1.52}$ for each population.

Mark the region identified in part (d) on each of the histograms below.
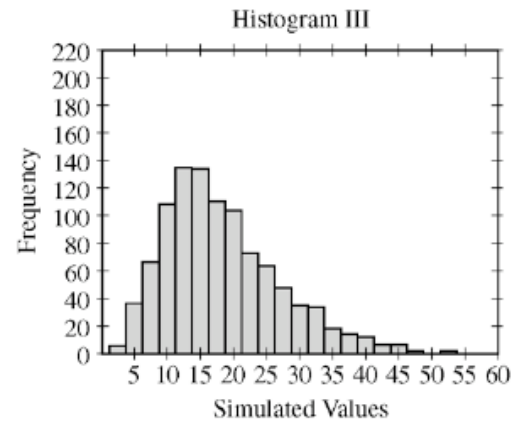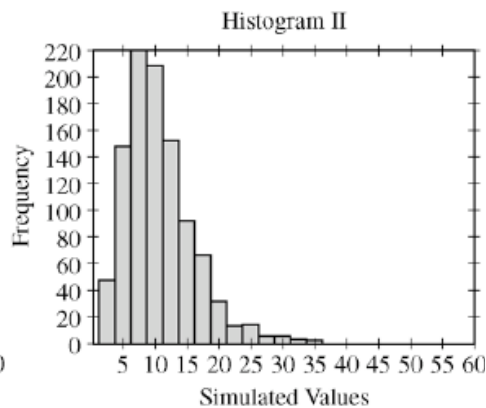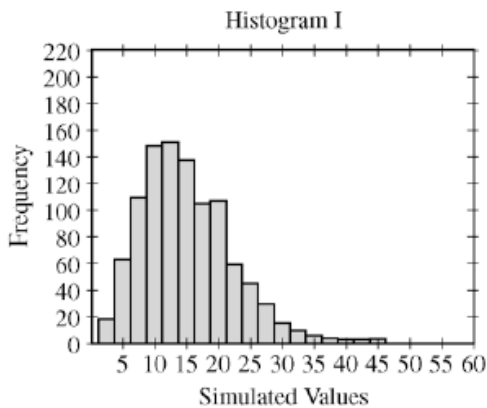
(f) Based on the regions that you marked in part (e), identify the simulated sampling distribution that corresponds to the population with the largest variance. Then identify the simulated sampling distribution that corresponds to the population with the smallest variance. Justify your choices.

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Total:___/4