

BINOMIAL PROBABILITY

THE EASY WAY!!

Binomial Probability

- Frequently used in analyzing and setting up surveys
- Our interest is in a **binomial random variable X**, which is the count of successes in *n trials*. The probability distribution of X is the binomial distribution.

A **binomial experiment** has the following assumptions:

- **SUCCESS OR FAILURE** – all observations are divided into two possible outcomes –
That's BI-nomial!
- **NUMBER** of observations is fixed.
- **INDEPENDENT OBSERVATIONS** --- knowledge of the outcomes of earlier trials does not affect the probability of success of the next trial
- **PROBABILITY** of success is constant

Calculating Binomial Probability

The Formula:

$$P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$$

where: $\binom{n}{k} = \frac{n!}{k!(n-k)!}$

The Calculator Functions:

- Binomial function for calculating a single value for the random variable X: $P(X = x)$
[2nd][VAR] [Binompdf](n, p, x) *Be sure to define all numbers in your calculator function!!*
- Binomial function for calculating a range of values to the left of X: $P(X \leq x)$
[2nd][VAR] [Binomcdf] (n, p, x)

The Complement Rule Method: (Using your calculator)

$$P(X > x) = 1 - P(X \leq x) = 1 - \text{binomcdf}(n, p, x)$$

Cumulative Binomial probability – the Easy Way!

- ALL Binomial calculations are done using: **[2nd][VAR] [Binompdf](n, p, x)**
- Calculating a probability for any range of values, we will add one step prior to the [Binompdf]
 - On your calculator select **[MATH][0]** which is the summation function: Σ
 - The number of trials and the probability of success will vary by problem...but x will always be **[X,T,θ,n]**

$$\sum_{x=\text{lower value}}^{\text{upper value}} \text{binompdf}(\text{\#trials}, P(\text{success}), \boxed{X, T, \theta, n})$$



THINK & DEFINE:

n = # of trials

p = probability of success

x = Interval : (Lower value, Upper value) specified in problem.

Example: Shoot a basketball 20 times from various distances on the court, given $P(\text{basket}) = 0.40$. Let $X =$ number of shots made.

1. Find the probability of making exactly 8 baskets:

$$P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$$

THiNK & DEFiNE:

$n = 20$
 $p = 0.40$
 $x = 8$



By Formula:

$$P(x = 8) = \binom{20}{8} \cdot 4^8 \cdot 6^{(20-8)}$$

By Calculator:

$$P(x = 8) = \text{Binompdf}(n = 20, p = 0.4, x = 8)$$

2. Find the probability of making fewer than 8 shots:

THiNK & DEFiNE:

$n = 20$
 $p = 0.40$
 x interval: $[0, 7]$



By Formula:

$$P(x < 8) = P(x \leq 7) = \binom{20}{0} \cdot 4^0 \cdot 6^{(20-0)} + \binom{20}{1} \cdot 4^1 \cdot 6^{(20-1)} + \dots + \binom{20}{7} \cdot 4^7 \cdot 6^{(20-7)} = 0.4159$$

By Calculator:

$$P(x \leq 7) = \sum_{x=0}^7 \text{Binompdf}(n = 20, p = 0.4, x = x) = 0.4159$$

3. Find the probability of making more than 12 shots:

THiNK & DEFiNE:

$n = 20$
 $p = 0.40$
 x interval: $[13, 20]$



By Formula:

$$P(x > 12) = P(x \geq 13) = \binom{20}{13} \cdot 4^{13} \cdot 6^{(20-13)} + \binom{20}{14} \cdot 4^{14} \cdot 6^{(20-14)} + \dots + \binom{20}{20} \cdot 4^{20} \cdot 6^{(20-20)} = 0.0210$$

By Calculator:

$$P(x \geq 13) = \sum_{x=13}^{20} \text{Binompdf}(n = 20, p = 0.4, x = x) = 0.0210$$



Whew!!
Lots of
Work!!



** Notice that this methodology is the same as #2 but with interval now starts at 13 and ends at 20*

Try it #1: In Roulette, 18 of the 38 spaces on the wheel are black. Suppose you observe the next 10 spins of a roulette wheel. Make sure to define the variable and distribution!!

1. What is the probability that exactly half of the spins land on black?



THINK & DEFINE:

Number of trials: $n = \underline{\hspace{2cm}}$
 Probability of success: $p = \underline{\hspace{2cm}}$
 Number of successes: $x = \underline{\hspace{2cm}}$

$$n = 10$$

$$p = \frac{18}{38} \text{ binompdf}\left(10, \frac{18}{38}, 5\right) = 0.2427$$

$$x = 5$$

2. What is the probability that at least 8 of the spins land on black?



THINK & DEFINE:

Number of trials: $n = \underline{\hspace{2cm}}$
 Probability of success: $p = \underline{\hspace{2cm}}$
 Number of successes: $x = \underline{\hspace{2cm}}$

$$n = 10$$

$$p = \frac{18}{38} \sum_{x=8}^{10} \text{binompdf}\left(10, \frac{18}{38}, x\right) = 0.0385$$

$$x \geq 8$$

* Is it OK to use the **binompdf** and **binomcdf** commands on the AP exam?

Yes! But **only** if you define your variables explicitly!

Try it #2: On the SAT, there are five answer choices (A, B, C, D, and E). The probability of randomly guessing the correct answer is .2.

- a. What is the probability that on a 25-question section of the SAT by complete random guessing that exactly 8 questions will be answered correctly?

$$P(\text{\#correct} = 8) = \text{Binomialpdf} (n = 25, p = .2, x = 8) = 0.0623$$

THINK & DEFINE:
 $n = 25$
 $p = 0.20$
 x interval: ??

- b. What is the probability that on a 25-question section of the SAT by complete random guessing that 6 or fewer questions will be answered correctly?

$$P(\text{\#correct} \leq 6) = \text{Binomialpdf} (n = 25, p = .2, x = 8)$$

$$P(x \leq 6) = \sum_{x=0}^6 \text{Binompdf} (n = 25, p = 0.2, x = x) = 0.780$$

- c. What is the probability that on a 25-question section of the SAT by complete random guessing that more than 8 questions will be answered correctly?

Think & Define: $n = 25, p = 0.20, x \in [9, 25]$

$$P(x \geq 9) = \sum_{x=9}^{25} \text{Binompdf} (n = 25, p = 0.2, x = x) = 0.0468$$

- d. What is the probability that on a 25-question section of the SAT by complete random guessing that more than 5 and less than 18 questions will be answered correctly?

Think & Define: $n = 25, p = 0.20, x \in [6, 17]$

$$P(6 \leq x \leq 17) = \sum_{x=6}^{17} \text{Binompdf} (n = 25, p = 0.2, x = x) = 0.3833$$

Binomial Probability Practice

- Major universities claim that 72% of their senior athletes graduate that year. Fifty (50) senior athletic students attending major universities are randomly selected and recorded in order of selection.
 - What is the probability that exactly 40 senior athletic students graduate that year?
 - What is the probability that 40 or 41 or 42 senior athletic students graduated that year?
 - What is the probability that 40 or fewer senior athletic students graduated that year?
 - What is the probability that 41 or more senior athletic students graduated that year?
 - What is the probability that 40 or more senior athletic students graduated that year?
- Will Fumble is the only receiver for the football team with the likelihood of catching a pass of .15.
 - What is the probability that 2 passes are caught out of 6 passes?
 - What is the probability that no passes are caught out of 6 passes?
 - What is the probability that only 0 or 1 pass is caught out of 6 passes?
 - What is the probability that 2 or fewer passes are caught out of 6 passes?
 - What is the probability that more than 2 passes are caught out of 6 passes?
 - What is the probability that he will catch between 3 and 5 passes out of the 6?
- The Telektronic Corp. purchases large shipments of fluorescent bulbs and uses this quality assurance plan: Randomly select and test 24 bulbs, then accept the shipment only if there is only one or fewer fail to work. If a particular shipment of thousands of bulbs actually has a 4% defect rate, what is the probability that this whole shipment is accepted?
- The Hemingway Financial Company prepares tax returns for individuals. According to the IRS, individuals making \$35,000 – \$50,000 are audited at a rate of 1%. The Hemingway company prepares 24 tax returns for individuals in that tax bracket, and 7 of them are audited.
 - Find the probability that exactly 7 returns of the 24 prepared will be audited.
 - Find the probability that 5 or 6 or 7 of the returns will be audited.
 - Based on your results, what can you conclude about the Hemingway customers? Are they just lucky or unlucky?
- The CBS TV show *60 Minutes* has been successful for many years. That show recently had a share of 20, meaning that among the TV sets in use, 20% were tuned to *60 Minutes* during its time slot (based on data from Nielsen Media Research). Assume that an advertiser wants to verify that 20% share value by conducting its own survey, and a pilot survey begins with 10 households with TV sets in use during the *60 Minutes* time slot.
 - Find the probability that at least one household is tuned to *60 Minutes*.
 - Find the probability that none of the households are tuned to *60 Minutes*.
 - Find the probability that at most two households are tuned to *60 Minutes*.
 - Find the probability that between two & five of the households are tuned to *60 Minutes*.
 - Based on your results, do you believe the claim made by the Nielsen Ratings?

6. According to a recent Census Bureau report, 12.7% of Americans live below the poverty level. Suppose you plan to sample at random 100 Americans and count the number of people who live below the poverty level.
 - a. What is the probability that you count exactly 10 in poverty?
 - b. What is the probability that you count 10 or less in poverty?
 - c. What is the probability that you count between 9 and 15 in poverty?
 - d. What is the probability that you count more than 15 in poverty?

7. A game is played with a spinner on a circle, like the minute hand on a clock. The circle is marked evenly from 1 to 100. The player spins the spinner and the resulting number is the number of seconds the player is given to solve a randomly selected mathematics problem. Suppose there are 30 students playing in the class.
 - a. What is the probability that 10 of the students received over a minute to solve the problem?
 - b. What is the probability that exactly half of the students received 30 or less seconds?
 - c. What is the probability that the four to ten receive more than a minute?
 - d. What is the probability that at least half of the students received 45 seconds or more?
 - e. What is the probability that a minute or more was received by the 10 students?

8. An agent sells life insurance policies to five equally aged, healthy people. According to recent data, the probability of a person living for 30 years or more is $\frac{2}{3}$. Calculate the probability that after 30 years:
 - a. All five people are still living.
 - b. At most three people are still living.
 - c. Exactly two people are still living.

9. It has been determined that 5% of drivers checked at a road stop show traces of alcohol and 10% of drivers checked do not wear seat belts. In addition, it has been observed that the two infractions are independent from one another. If an officer stops five drivers at random:
 - a. Calculate the probability that at least one of the drivers checked has committed at least one of the two offenses.
 - b. Calculate the probability that exactly three of the drivers have committed at least one of the two offenses.

10. A pharmaceutical lab states that a drug causes negative side effects in 3 of every 100 patients. To confirm this affirmation, another laboratory chooses 5 people at random who have consumed the drug. What is the probability of the following events?
 - a. None of the five patients experience side effects.
 - b. At least two experience side effects.
 - c. Fewer than three experience side effects.
 - d. What is the average number of patients that the laboratory should expect to experience side effects if they choose 100 patients at random?

11. An instructor develops a True-False exam for an introductory stat course. A student, who didn't study, randomly guesses on each question. Making the reasonable assumption that the probability of success (guessing correctly) on each question is equal to .5, answer the following questions.
- If the exam has 20 questions, use the probability tool to find the probability that the student will guess exactly 5 right? Exactly 11 right? Exactly 19 right?
 - If there are 20 questions, what is the probability that the student guess correctly between 7 and 12 times?
 - What is the probability that the student will score less than 7 or more than 12 correct?
 - What is the probability that the student will score 18 or above?
 - What is the probability that the student will score 14 or above?
 - What single score is the most likely? And what is the probability of obtaining this score?
 - If there is only one question ($N = 1$), what is the probability of getting 100% by guessing?
 - If there are two questions, what is the probability of getting 100%?
 - If there are five questions, what is the probability of getting 100%?
 - If there are 10 questions, what is the probability of getting 100%?
12. At the State Fair there is a booth where people can throw dimes onto a table that has dishes on it. Suppose that the chance that a dime lands on a dish is equal to .3, that is, $p = .3$. Suppose for a dollar you can buy 10 throws.
- The most desirable prizes (on the highest shelf) require that you get 9 or 10 (out of 10 dimes) in dishes. What is the probability of getting 9 or 10 out of 10?
 - The second highest requires that you get 7 or 8 dimes in dishes. What are your chances of doing that?
 - The third highest shelf requires that you get either 5 or 6 dimes in dishes. What are your chances of doing that?

Binomial Probability Practice - KEY

- Major universities claim that 72% of their senior athletes graduate that year. Fifty (50) senior athletic students attending major universities are randomly selected and recorded in order of selection.
 - What is the probability that exactly 40 senior athletic students graduate that year? **.0598**
 - What is the probability that 40 or 41 or 42 senior athletic students graduated that year? **0.1179**
 - What is the probability that 40 or fewer senior athletic students graduated that year? **0.9260**
 - What is the probability that 41 or more senior athletic students graduated that year? **0.0739**
 - What is the probability that 40 or more senior athletic students graduated that year? **0.1337**
- Will Fumble is the only receiver for the football team with the likelihood of catching a pass of .15.
 - What is the probability that 2 passes are caught out of 6 passes? **0.1762**
 - What is the probability that no passes are caught out of 6 passes? **0.3771**
 - What is the probability that only 0 or 1 pass is caught out of 6 passes? **0.7765**
 - What is the probability that 2 or fewer passes are caught out of 6 passes? **0.9527**
 - What is the probability that more than 2 passes are caught out of 6 passes? **0.0473**
 - What is the probability that he will catch between 3 and 5 passes out of the 6? **0.0473**
- The Telektronic Corp. purchases large shipments of fluorescent bulbs and uses this quality assurance plan: Randomly select and test 24 bulbs, then accept the shipment only if there is only one or fewer fail to work. If a particular shipment of thousands of bulbs actually has a 4% defect rate, what is the probability that this whole shipment is accepted? **0.7508**
- The Hemingway Financial Company prepares tax returns for individuals. According to the IRS, individuals making \$35,000 – \$50,000 are audited at a rate of 1%. The Hemingway company prepares 24 tax returns for individuals in that tax bracket, and 7 of them are audited.
 - Find the probability that exactly 7 returns of the 24 prepared will be audited. **0.000000003**
 - Find the probability that 5 or 6 or 7 of the returns will be audited. **0.0000036**
 - Based on your results, what can you conclude about the Hemingway customers? Are they just lucky or unlucky? **They are very unlucky or Hemingway is really in the business of writing great fiction!!**
- The CBS TV show *60 Minutes* has been successful for many years. That show recently had a share of 20, meaning that among the TV sets in use, 20% were tuned to *60 Minutes* during its time slot (based on data from Nielsen Media Research). Assume that an advertiser wants to verify that 20% share value by conducting its own survey, and a pilot survey begins with 10 households with TV sets in use during the *60 Minutes* time slot.
 - Find the probability that at least one household is tuned to *60 Minutes*. **0.8926**
 - Find the probability that none of the households are tuned to *60 Minutes*. **0.1074**
 - Find the probability that at most two households are tuned to *60 Minutes*. **0.6778**
 - Find the probability that between two & five of the households are tuned to *60 Minutes*. **0.6178**
 - Based on your results, do you believe the claim made by the Nielsen Ratings? **Yes, the probability of having 2 of the ten households or fewer tuned to 60 Minutes is 67.78% which is not unusual.**

6. According to a recent Census Bureau report, 12.7% of Americans live below the poverty level. Suppose you plan to sample at random 100 Americans and count the number of people who live below the poverty level.
- What is the probability that you count exactly 10 in poverty? 0.0928
 - What is the probability that you count 10 or less in poverty? 0.2614
 - What is the probability that you count between 9 and 15 in poverty? 0.7046
 - What is the probability that you count more than 15 in poverty? 0.1969
7. A game is played with a spinner on a circle, like the minute hand on a clock. The circle is marked evenly from 1 to 100. The player spins the spinner and the resulting number is the number of seconds the player is given to solve a randomly selected mathematics problem. Suppose there are 30 students playing in the class.
- What is the probability that 10 of the students received over a minute to solve the problem? 0.1152
 - What is the probability that exactly half of the students received 30 or less seconds? 0.0106
 - What is the probability that the four to ten receive more than a minute? 0.2912
 - What is the probability that at least half of the students received 45 seconds or more? 0.8016
 - What is the probability that a minute or more was received by 10 or more students? 0.8237
8. An agent sells life insurance policies to five equally aged, healthy people. According to recent data, the probability of a person living for 30 years or more is $\frac{2}{3}$. Calculate the probability that after 30 years:
- All five people are still living. 0.1317
 - At most three people are still living. 0.5391
 - Exactly two people are still living. 0.1646
9. It has been determined that 5% of drivers checked at a road stop show traces of alcohol and 10% of drivers checked do not wear seat belts. In addition, it has been observed that the two infractions are independent from one another. If an officer stops five drivers at random:
- Calculate the probability that at least one of the drivers checked has committed at least one of the two offenses.
0.5431
 - Calculate the probability that exactly three of the drivers have committed at least one of the two offenses.
0.3344
10. A pharmaceutical lab states that a drug causes negative side effects in 3 of every 100 patients. To confirm this affirmation, another laboratory chooses 5 people at random who have consumed the drug. What is the probability of the following events?
- None of the five patients experience side effects. 0.8587
 - At least two experience side effects. 0.0085
 - Fewer than three experience side effects. 0.9997
 - What is the average number of patients that the laboratory should expect to experience side effects if they choose 100 patients at random? 3

11. An instructor develops a True-False exam for an introductory stat course. A student, who didn't study, randomly guesses on each question. Making the reasonable assumption that the probability of success (guessing correctly) on each question is equal to .5, answer the following questions.
- If the exam has 20 questions, use the probability tool to find the probability that the student will guess exactly 5 right? Exactly 11 right? Exactly 19 right? **0.0148, 0.1602, 0.000019**
 - If there are 20 questions, what is the probability that the student guess correctly between 7 and 12 times? **0.8108**
 - What is the probability that the student will score less than 7 or more than 12 correct? **0.1892**
 - What is the probability that the student will score 18 or above? **0.000201**
 - What is the probability that the student will score 14 or above? **0.0577**
 - What single score is the most likely? And what is the probability of obtaining this score?
To see the distribution quickly, put numbers 1 – 20 into L_1 and the function $\text{binompdf}(20, .5, L_1)$ into the L_2 header $P(x = 10) = 0.1762$
 - If there is only one question ($N = 1$), what is the probability of getting 100% by guessing? **0.5**
 - If there are two questions, what is the probability of getting 100%? **0.25**
 - If there are five questions, what is the probability of getting 100%? **0.03125**
 - If there are 10 questions, what is the probability of getting 100%? **0.000977**
12. At the State Fair there is a booth where people can throw dimes onto a table that has dishes on it. Suppose that the chance that a dime lands on a dish is equal to .3, that is, $p = .3$. Suppose for a dollar you can buy 10 throws.
- The most desirable prizes (on the highest shelf) require that you get 9 or 10 (out of 10 dimes) in dishes. What is the probability of getting 9 or 10 out of 10? **0.000144**
 - The second highest requires that you get 7 or 8 dimes in dishes. What are your chances of doing that? **0.0104**
 - The third highest shelf requires that you get either 5 or 6 dimes in dishes. What are your chances of doing that? **0.1397**