The Birthday Paradox:
A Monte Carlo Simulation Activity

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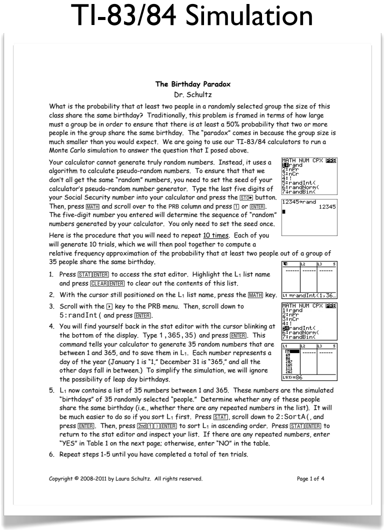
Birthday Paradox

How large must a group be in order to ensure that there is at least a 50% probability that two or more people in the group have the same birthday?

What is the probability that at least two people in your reading room have the same birthday?

TI-83/84 Simulation



The Birthday Paradox

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What is the probability that at least two people in a randomly selected group the size of this class share the same birthday? Traditionally, this problem is framed in terms of how large must a group be in order to ensure that there is at least a 50% probability that two or more people in the group share the same birthday. The "paradox" comes in because the group size is much smaller than you would expect. We are going to use our TI-83/84 calculators to run a Monte Carlo simulation to answer the question that I posed above.

Your calculator cannot generate truly random numbers. Instead, it uses a algorithm to calculate pseudo-random numbers. To ensure that that we don't all get the same "random" numbers, you need to set the seed of your calculator's pseudo-random number generator. Type the last five digits of your Social Security number into your calculator and press the STO▸ button. Then, press MATH and scroll over to the PRB column and press 1 or ENTER. The five-digit number you entered will determine the sequence of "random" numbers generated by your calculator. You only need to set the seed once.

Here is the procedure that you will need to repeat 10 times. Each of you will generate 10 trials, which we will then pool together to compute a relative frequency approximation of the probability that at least two people out of a group of 35 people share the same birthday.

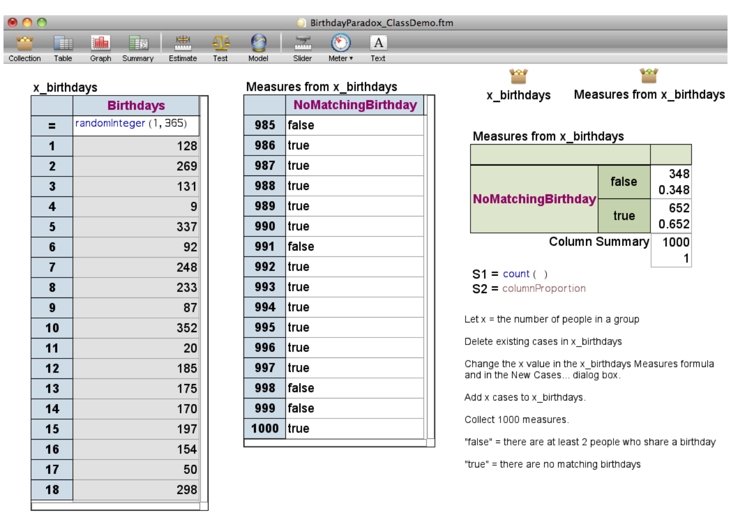
1. Press STAT ENTER to access the stat editor. Highlight the L_1 list name and press CLEAR ENTER to clear out the contents of this list.
2. With the cursor still positioned on the L_1 list name, press the MATH key.
3. Scroll with the ▸ key to the PRB menu. Then, scroll down to `5:randInt(` and press ENTER.
4. You will find yourself back in the stat editor with the cursor blinking at the bottom of the display. Type `1,365,35)` and press ENTER. This command tells your calculator to generate 35 random numbers that are between 1 and 365, and to save them in L_1. Each number represents a day of the year (January 1 is "1," December 31 is "365," and all the other days fall in between.) To simplify the simulation, we will ignore the possibility of leap day birthdays.
5. L_1 now contains a list of 35 numbers between 1 and 365. These numbers are the simulated "birthdays" of 35 randomly selected "people." Determine whether any of these people share the same birthday (i.e., whether there are any repeated numbers in the list). It will be much easier to do so if you sort L_1 first. Press STAT, scroll down to `2:SortA(`, and press ENTER. Then, press 2nd 1) ENTER to sort L_1 in ascending order. Press STAT ENTER to return to the stat editor and inspect your list. If there are any repeated numbers, enter "YES" in Table 1 on the next page; otherwise, enter "NO" in the table.
6. Repeat steps 1-5 until you have completed a total of ten trials.

(Worksheet available on StatsMonkey and my own website.)

FATHOM Simulation

BirthdayParadox_ClassDemo.ftm

(Fathom demo available on StatsMonkey and my own website.)



- Let A = event that no two people in this room have the same birthday.
- Then, $\bar{A}$ = event that two or more people share the same birthday.
- $P(\bar{A}) = 1 - P(A)$

- If there is only one person in the room, then the probability that no two people have the same birthday is 1.
- Also, the probability that the person's birthday is on one of the 365 days of the year is $\frac{365}{365} = 1$

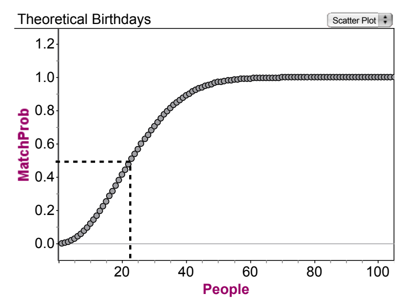
- If there are two people, the probability that they do not share the same birthday is $\frac{365}{365} \times \frac{364}{365} = 0.997$
- If there are three people, the probability that nobody shares a birthday with anybody else in the room is $\frac{365}{365} \times \frac{364}{365} \times \frac{363}{365} = 0.992$

- If there are n people in the room, then the probability that no two people share the same birthday is

$$P(A) = \frac{365}{365} \times \frac{364}{365} \times \frac{363}{365} \times \ldots \times \frac{365-n+1}{365}$$

- Then, the probability that two or more people out of n do share the same birthday is

$$P(\bar{A}) = 1 - P(A) = 1 - \left[\frac{365}{365} \times \frac{364}{365} \times \frac{363}{365} \times \ldots \times \frac{365-n+1}{365}\right]$$



People	MatchProb
1	0
2	0.00273973
3	0.00820417
4	0.0163559
5	0.0271356
6	0.0404625
7	0.0562357
8	0.0743353
9	0.0946238
10	0.116948
11	0.141141
12	0.167025
13	0.19441
14	0.223103
15	0.252901
16	0.283604
17	0.315008
18	0.346911
19	0.379119
20	0.411438
21	0.443688
22	0.475695
23	0.507297
24	0.538344
25	0.5687
26	0.598241
27	0.626859
28	0.654461
29	0.680969
30	0.706316
31	0.730455
32	0.753348
33	0.774972
34	0.795317
35	0.814383
36	0.832182

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