
Standard Normal Calculations



This activity can be used to illustrate practical uses of standard normal calculations and their interpretations. It is meant to reinforce the concept of normal distributions and individual observations as well as introduce students to the basic idea of inference. While students have not studied sampling distributions or inferential procedures, they should be able to use what they know about variability and normal distributions to make some basic inferential conclusions.

Materials Needed:

- 2 1.69 bags of *milk chocolate* m&m's
- At least 1 1.69 bag of *peanut butter* m&m's

Activity:

Students should be familiar with the concept of normal distributions and should have a basic understanding of sampling variability. Discuss the fact that the proportion of yellow m&m's in an individual bag will vary from bag to bag.

Note that the proportion of yellow m&m's in individual bags varies according to approximately $N(0.14, 0.05)$. Sketch the distribution, noting where each standard deviation falls. Discuss the Empirical (68-95-99.7) Rule as it relates to the proportion of yellow m&m's in a bag. Ask for student impressions of 'extreme' proportions. That is, at what point would they suspect the claim that 14% of m&m's are yellow? Why would they suspect it? Estimate the percent of bags that have less than 10% yellow m&m's, greater than 20% yellow, etc.

Open *milk chocolate* m&m's and note proportion of yellow candies. {Note: if this proportion is exactly 14%, you may wish to use another bag} Locate and plot this proportion on the normal curve. Calculate and interpret the standardized z-score and corresponding proportions above and below this value. If you have 2 bags, repeat the calculations and find the proportion of bags that would fall between the two observations.

After exhausting all possible calculations with the *milk chocolate* m&m's, open and find the proportion of yellow *peanut butter* m&m's. {Note: 20% of peanut butter m&m's are yellow, as compared to 14% of milk chocolate m&m's--do not tell students this}. Locate this proportion on the $N(0.14, 0.05)$ distribution and discuss whether or not they think yellow peanut butter m&m's are produced in the same proportion as yellow milk chocolate m&m's.



Are These m&m's "Normal" or Just "Plain"?

Standard Normal Calculations

We have observed variability in color distributions from bag to bag of "plain" milk chocolate m&m's. According to the m&m website, 14% of milk chocolate m&m's are yellow. Does that mean we are guaranteed 14% of the candies in each bag will be yellow? Should you be concerned if only 10% are yellow? What if all of them are? At what point would you suspect the advertised proportion? We will discuss each of these questions as we explore standard normal calculations with some sample bags of m&m's.

Background Information:

We know the proportion of yellow m&m's varies from bag to bag. Suppose these proportions follow an approximately normal distribution $N(0.14, 0.05)$. Sketch this distribution below and note 1, 2, and 3 standard deviations above and below the mean. Interpret the Empirical (68-95-99.7) Rule in the context of this situation.

Sample Information:

Our bag of m&m's contained _____ candies. There were _____ yellow m&m's.
 The sample proportion of yellow candies for our bag is _____ / _____ = _____.

Standard Normal Calculation:

Recall, a "z-score" is a value that tells us how many standard deviations above or below the mean a particular observation falls. To find this value, we must subtract the mean from our observation and divide the result by the standard deviation. That is,

$$z = \frac{x - \bar{x}}{s} = \frac{\boxed{} - \boxed{}}{\boxed{}} = \boxed{}$$

We can use this z-score to determine what percent of bags of m&m's (of the same size) would have a yellow proportion less than our observed proportion. Sketch two normal distributions for yellow proportions below and note our observed proportion on each curve. Using your z-table, determine the proportion of bags of the same size that would have fewer yellow candies. Shade this area on the first curve. On the second curve, shade and calculate the proportion of bags of the same size that would have more yellow candies.

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Suppose we had a second bag of m&m's. We would expect about 14% of the candies in the second bag would be yellow. However, like the first bag, there is a chance that proportion will not equal 0.14 (or the proportion in the first bag, for that matter). Use the proportions from the first bag and from a new bag to determine what percent of bags of m&m's of the same size will have a yellow proportion *between* those two values.

	Bag 1	Bag 2
Yellow Proportion		
z-score		
% Bags Below Observed		

Sketch the two observed proportions on the normal distribution $N(0.14, 0.05)$ and note the percent of observations we would expect to see *between* the two observed proportions.

What About Peanut Butter m&m's?

Do peanut butter m&m's follow the same color distribution as milk chocolate m&m's? If so, we would expect about 14% of the candies in a peanut butter m&m bag would be yellow. Would you be surprised if 15% were yellow? What about 20%? 30%? At what point would you suspect the color distribution for peanut butter m&m's may be different?

Open a bag of peanut butter m&m's and note the proportion of yellow: $\frac{\quad}{\quad} = \quad$

Plot this value on the $N(0.14, 0.05)$ distribution and calculate the % of observations we'd expect to be *more extreme* than this observation. Based on this %, do you feel you have evidence to suggest the color distribution of peanut butter m&m's may be different? Why or why not?